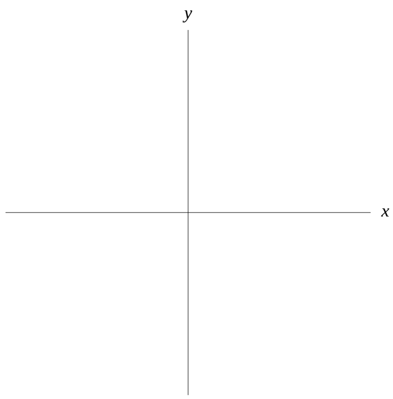
The Gradient and Level Sets

1. Let $f(x, y) = x^2 + y^2$.

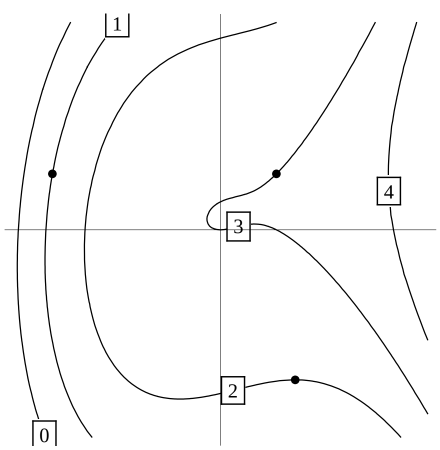
   (a) Find the gradient ∇f.

   (b) Pick your favorite positive number k, and let $\mathcal{C}$ be the curve $f(x, y) = k$. Draw the curve on the axes below. Now pick a point (a, b) on the curve $\mathcal{C}$. What is the vector $\nabla f(a, b)$? Draw the vector $\nabla f(a, b)$ with its tail at the point (a, b). What relationship does the vector have to the curve?



   (c) Let $\vec{r}(t)$ be any parameterization of your curve $\mathcal{C}$. What is $f(\vec{r}(t))$? What happens if you use the Chain Rule to find $\frac{d}{dt} f(\vec{r}(t))$? Use this to explain your observation from (b).

2. Here is the level set diagram (contour map) of a function $f(x, y)$. The value of $f(x, y)$ on each level set is labeled. For each of the three points (a, b) marked in the picture, draw a vector showing the direction of $\nabla f(a, b)$. (Don't worry about the magnitude of $\nabla f(a, b)$.)



3. Let S be the cylinder $x^2 + y^2 = 4$. Find the plane tangent to S at the point $(1, \sqrt{3}, 5)$.

4. Let S be the surface $z = y \sin x$. Find the plane tangent to S at the point $\left(\frac{\pi}{6}, 2, 1\right)$.

5. Suppose that $3x + 4y - 5z = -4$ is the plane tangent to the graph of $f(x, y)$ at the point $(1, 2, 3)$.

   (a) Find $\nabla f(1, 2)$.

   (b) Use linear approximation to approximate $f(1.1, 1.9)$.